

Massive Boson-Fermion Degeneracy and the Early Structure of the Universe

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Abstract

The existence of a new kind of *massive boson-fermion symmetry* is shown explicitly in the framework of the heterotic, type II and type IIA orientifold superstring theories. The target space-time is two-dimensional. Higher dimensional models are defined via large marginal deformations of $J\bar{J}$ -type. The spectrum of the initial undeformed two dimensional vacuum consists of massless boson degrees of freedom, while all massive boson and fermion degrees of freedom exhibit a new Massive Spectrum Degeneracy Symmetry (*MSDS*). This precise property, distinguishes the *MSDS* theories from the well known supersymmetric *SUSY*-theories. Some proposals are stated in the framework of these theories concerning the structure of:

- (i) The Early Non-singular Phase of the Universe,
- (ii) The two dimensional boundary theory of AdS_3 Black-Holes,
- (iii) Plausible applications of the *MSDS* theories in particle physics, alternative to *SUSY*.

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1 Introduction

In string theories the origin of space-time supersymmetry in 1+9 dimensions follows from the *equivalence* of the vector- V_8 and spinor- S_8 of the underlying $SO(8)$ “helicity” group (light-cone picture) [1]. In lower dimensions, depending on the choice of the internal compactified space, some of the space-time supersymmetries are broken. The initial $SO(8)$ in 1+9 dimensions is broken by compactification to $SO(d-2) \times G$, (Helicity \oplus R -symmetry group). The R -symmetry group G characterizes the internal compactified space and ensures in lower space-time dimensions the number of the unbroken supersymmetries N_d [2], as well as a boson-fermion spectrum degeneracy.

In the string framework, G is decomposed in left- and right- moving factors $G_L \times G_R$. It turns out that the number of space-time supersymmetry depends on both G_L and G_R since both can induce in space-time “boson-fermion spectral flow degeneracies”. In the world-sheet G_L and G_R are expressed in terms of left- and right- moving superconformal symmetries. I am stressing here the well-known correspondence between the number of space-time supersymmetries, N_d , and the superconformal symmetries $\mathcal{N}_{(L,R)}$ of the string world-sheet [2], like for instance:

(i) The existence of $N_4 = 1$ supersymmetry in the heterotic, requiring $G_L = U(1)$ with $\mathcal{N}_{(L,R)} = (2, 0)$ superconformal symmetry in the world-sheet [2] [3].

(ii) When $G_L = U(2)$ in the heterotic, the superconformal symmetry is at least $\mathcal{N} = (4, 0)$ and the space-time supersymmetry becomes $N_4 = 2$.

(iii) In type II theories supersymmetry emerges from left- and right- moving sectors; then the space-time supersymmetry is : $N_4 = 1$ when $\mathcal{N} = (2, 1)$; $N_4 = 2$ when $\mathcal{N} = (4, 1)$ or $\mathcal{N}_{(L,R)} = (2, 2)$; $N_4 = 4$ when $\mathcal{N}_{(L,R)} = (4, 4)$,

(iv) The maximal supersymmetry “in any type of superstring theory” is achieved via toroidal compactifications T^n , $n = 10 - d$, leading to $N_4 = 4$ in heterotic or Type I and to $N_4 = 8$ in type IIA or IIB superstrings.

In our days, there are well known procedures to reduce the number of supersymmetries, namely: via symmetric orbifolds [4] (\equiv Calabi-Yau [1]) compactification, via fermionic [5] [6] and Gepner constructions [3], asymmetric orbifolds [7] (\equiv generalized CY with torsion [9]),

or type II orientifold compactifications [8] with or without geometrical [10] [11] or non-geometrical [12] fluxes.

The main achievement of the present work is the discovery of a *new correspondence* between:

- *A Massive boson-fermion Spectrum-Degeneracy Symmetry in space-time, MSDS*
- *2d-world-sheet Spectral-Flow Super-Conformal Symmetry $SFSC_{\frac{3}{2}}$*

The initial construction assumes a two dimensional target space-time where the remaining extra eight internal dimensions are compact with a typical scale the string scale. The physical motivation of these 2-d constructions is suggested by the “Big-Bang” picture of general relativity that predicts an initial classical singularity at the early times of the universe. Assuming a compact space, close to the singularity, the typical scale of the universe reaches at early times the gravitational scale (string scale). Obviously at this early epoch the classical gravity is not valid any more and has to be replaced by a more fundamental singularity-free theory.

In the framework of superstrings, some special theories in $d \leq 2$ exist, which are able to describe successfully the early phase of the universe. These theories are non-singular and are based on a Spectral-Flow Super-Conformal Symmetry, $SFSC_{\frac{3}{2}}$, in the world-sheet. In the space-time, the spectrum of the massive bosons and fermions shows a Massive Spectrum Degeneracy Symmetry, *MSDS*.

The other important point which is in favor to my proposal comes from the analysis of a phase transition above the Hagedorn temperature [13] $T \geq T_H$. It is well known that for high temperatures, $T > T_H$, the string partition function diverges due to the a thermal winding state which becomes tachyonic [14] [15] [16]. This is a signal of a phase transition towards a new vacuum. Many proposals were made in the literature concerning the high temperature non-singular phase [14], [16] [17]. As I will argue in this work, the *MSDS*-vacua are potential candidates, able to describe the early phase of the universe above T_H .

2 The maximally symmetric *MSDS* Vacuum

As was stated in the introduction, in the early stage of the universe all nine, or at least the eight- space coordinates are assumed to be compact and closed to the string scale. Furthermore, the supersymmetric vacuum cannot be the desire one since in any non-trivial cosmological or thermal background the space-time supersymmetry is always broken. The above two statements can be easily formulated in terms of the fermionic construction [5] [6] where all 2d-world-sheet compact space coordinates are expressed in terms of free 2d-fermions. The advantage of this construction is due to a consistent separation of the left- and right-moving world-sheet degrees of freedom in terms of left- and right-moving 2d fermions which give us the possibility in manipulating easily the left-right asymmetric and even non-geometrical constructions of vacua in string theory.

2.1 The type II *MSDS* Vacuum

The starting example is in the type II theories where the left- and right-moving degrees of freedom are:

- The light-cone degrees: $(\partial X^0, \Psi^0), (\partial X^L, \Psi^L)$
- The super-reparametrization ghosts: $(b, c) (\beta, \gamma)$
- The transverse super-coordinates $(\partial X^I, \Psi^I), I = 1, \dots, 8$.

In the fermionic construction the transverse super-coordinates are replaced by a set of free fermions in the adjoint representation of a semi-simple gauge group H [18] [5], $\{\chi^a\}$, $a = 1, \dots, n$, $n = \dim[H] = 24$. The simplest choice of H is:

$$H = SU(2)^8 \equiv SO(4)^4, \quad (2.1)$$

where the transverse super-coordinates, $(\partial X^I, \Psi^I)$ are replaced by (y^I, w^I, Ψ_I) so that for every $I = 1, \dots, 8$, the coordinate currents $i\partial X^I = y^I w^I$ are expressed in terms of y^I, w^I 2d world-sheet fermions. For every I , $\{y^I, w^I, \Psi_I\}$ define the adjoint representation, of $SU(2)_{k=2}$. The choice $H = SU(2)^8$ of the coordinate-fermionisation is not unique. Other

non-trivial choices of the coordinate-fermionisation exist based on different H :

$$H = SU(5), \quad H = SO(7) \times SU(2), \quad H = G_2 \times Sp(4), \quad H = SU(4) \times SU(2)^3, \quad H = SU(3)^3. \quad (2.2)$$

In all above choices the dimension of H is always equal to 24, which as we will see it will be one of the necessary conditions for the realization of the $MSDS$ symmetry. Once the choice of boundary conditions on world-sheet respect the global existence of the H symmetry, then H is promoted to a local gauge symmetry on the target space-time [18] [19].

The fundamental operators are as usual the left- and right- moving energy-momentum tensor T_B with conformal dimension $h_B = 2$ and the superconformal operator T_F with $h_F = 3/2$. T_F is responsible for the local $\mathcal{N} = 1$ world-sheet superconformal symmetry [1].

$$T_B = -\frac{1}{2}(\partial X_0)^2 - \frac{1}{2}\Psi_0\partial\Psi_0 + \frac{1}{2}(\partial X_1)^2 + \frac{1}{2}\Psi_1\partial\Psi_1 + \sum_{a=1}^{24} \frac{1}{2} \chi^a\partial\chi^a$$

$$T_F = i\partial X_0\Psi_0 + i\partial X_1\Psi_1 + f_{abc} \chi^a\chi^b\chi^c \quad (2.3)$$

where f_{abc} are the structure constants of the group H .

Following the rules of the fermionic construction and respecting the H -symmetry we can construct a very special tachyon free vacuum, with left- right- holomorphic factorization of the partition function:

$$Z_{II}^{2d} = \int_{\mathcal{F}} \frac{d^2\tau}{(\text{Im}\tau)^2} \left[\frac{1}{2} \sum_{a_L, b_L} (-)^{a_L+b_L} \frac{\theta \left[\begin{smallmatrix} a_L \\ b_L \end{smallmatrix} \right]^{12}}{\eta^{12}} \right] \left[\frac{1}{2} \sum_{a_R, b_R} (-)^{a_R+b_R} \frac{\bar{\theta} \left[\begin{smallmatrix} a_R \\ b_R \end{smallmatrix} \right]^{12}}{\bar{\eta}^{12}} \right]$$

or

$$Z_{II}^{2d} = \int_{\mathcal{F}} \frac{d^2\tau}{(\text{Im}\tau)^2} \left[\frac{(\theta_3^{12} - \theta_4^{12}) - (\theta_2^{12} - \theta_1^{12})}{2\eta^{12}} \right] \left[\frac{(\bar{\theta}_3^{12} - \bar{\theta}_4^{12}) - (\bar{\theta}_2^{12} - \bar{\theta}_1^{12})}{2\bar{\eta}^{12}} \right] \quad (2.4)$$

or in terms of the $SO(24)$ characters:

$$Z_{II}^{2d} = \int_{\mathcal{F}} \frac{d^2\tau}{(\text{Im}\tau)^2} \left[V^{(24)} - S^{(24)} \right] \left[\bar{V}^{(24)} - \bar{S}^{(24)} \right] \quad (2.5)$$

The above expressions of Z_{II}^{2d} remain the same for any choice of left-and right-moving H -group, H_L , H_R . In this respect Z_{II}^{2d} is a unique tachyon free partition function, (modulo the

chirality of the left- and right-spinors), *that respects the $H_L \times H_R$ gauge symmetry*.

The holomorphic factorization of Z_{II}^{2d} reminds us the corresponding supersymmetric partition function in two dimensions obtained via T^8 compactification in two space-time dimensions with maximal supersymmetry:

$$Z_{SUSY}^{2d} = \int_{\mathcal{F}} \frac{d^2\tau}{(\text{Im}\tau)^2} \left\{ \frac{\Gamma_{(8,8)}}{\eta^8 \bar{\eta}^8} \right\} [V^{(8)} - S^{(8)}] [\bar{V}^{(8)} - \bar{S}^{(8)}] \quad (2.6)$$

Modulo the non-holomorphic Narain lattice factor, $\left\{ \frac{\Gamma_{(8,8)}}{\eta^8 \bar{\eta}^8} \right\}$ in Z_{SUSY}^{2d} , the two partition functions share holomorphic factorization properties:

- Z_{SUSY}^{2d} is zero due to the equivalence of the bosonic $V^{(8)}$ and the fermionic $S^{(8)}$ $SO(8)$ characters. This spectacular property follows from the θ -function identities [1]:

$$\theta_3^4 - \theta_4^4 - \theta_2^4 = 0, \quad \theta_1^4 = 0, \quad (2.7)$$

having their origin to the triality property of $SO(8)$ affine Lie algebra, namely, the equivalence of the vector $V^{(8)}$, the spinor $S^{(8)}$ and the conjugate spinor $C^{(8)}$ representations at any massive level.

- Contrary to SUSY Z_{II}^{2d} is not zero. This is a signal of a non-equivalent bosonic and fermionic mass spectrum. However, the situation is even more spectacular than the supersymmetric case! Thanks to the $SO(24)$ affine algebra [20],

$$V^{(24)} - S^{(24)} = \text{constant} \equiv 24 \quad (2.8)$$

This follows from the theta function identities

$$\theta_3^4 - \theta_4^4 - \theta_2^4 = 0, \quad \theta_1^4 = 0, \quad \theta_2\theta_3\theta_4 = 2\eta^3$$

that imply the identity

$$\frac{1}{2} \left(\frac{\theta_3^{12}}{\eta^{12}} - \frac{\theta_4^{12}}{\eta^{12}} \right) - \frac{1}{2} \left(\frac{\theta_2^{12}}{\eta^{12}} - \frac{\theta_1^{12}}{\eta^{12}} \right) = 24 \quad (2.9)$$

- The above identity shows that the spectrum of massive bosons and massive fermions is

identical to all string mass levels ! This is similar to the property of supersymmetric theories. In the massless level however the situation is different: Although there are 24 left-moving bosonic degrees of freedom there are no massless fermionic states. Similarly there are 24 right-moving bosonic states. In total there are $24 \times 24 = 576$ scalar bosons at the massless level in the adjoint representation of H_L and H_R .

- The integrated partition function is thus equal to $576 \times \mathcal{I}$, where \mathcal{I} is the integral over the fundamental domain

$$\mathcal{I} = \int_{\mathcal{F}} \frac{d^2\tau}{(\text{Im}\tau)^2} = \frac{\pi^2}{3}, \quad Z_H^{2d} = \frac{\pi^2}{3} \times 576.$$

- In SUSY models the underlying global gauge symmetry $H_L \times H_R$ is necessarily broken to a discrete sub-group contrary to the *MSDS* model where $H_L \times H_R$ is unbroken, and is promoted to a local space-time gauge symmetry.

In the next section I will examine in more detail the origin of the boson-fermion massive spectral symmetry, *MSDS*, and its difference from the conventional supersymmetry.

2.2 Chiral superconformal algebra and spectral flows in *MSDS*

The symmetry operators of the *MSDS* vacuum are the usual holomorphic (anti-holomorphic) operators T_B, T_F (\bar{T}_B, \bar{T}_F) giving rise to the standard $\mathcal{N} = (1, 1)$ world-sheet superconformal symmetry induced by the familiar Operator Product Expansion [1] (OPE) with $\hat{c} = \frac{2}{3}c = 8$.

$$\begin{aligned} T_B(z) T_B(w) &\sim \frac{3\hat{c}}{4(z-w)^4} + \frac{2T_B(w)}{(z-w)^2} + \frac{\partial T_B(w)}{(z-w)} \\ T_B(z) T_F(w) &\sim \frac{3T_F(w)}{2(z-w)^2} + \frac{\partial T_F(w)}{(z-w)}, \quad T_F(z) T_F(w) \sim \frac{\hat{c}}{(z-w)^3} + \frac{2T_B(w)}{(z-w)} \end{aligned} \quad (2.10)$$

The extra symmetry operators are the H -currents: $J^a \equiv f_{bc}^a \chi^b \chi^c$ and χ^a with conformal weights $h_J = 1$ and $h_\chi = \frac{1}{2}$ respectively.

$$\begin{aligned} T_B(z) J^a(w) &\sim \frac{J^a(w)}{(z-w)^2} + \frac{\partial J^a(w)}{(z-w)}, & T_B(z) \chi^a(w) &\sim \frac{\chi^a(w)}{2(z-w)^2} + \frac{\partial \chi^a(w)}{(z-w)}, \\ T_F(z) J^a(w) &\sim \frac{\chi^a}{(z-w)^2} + \frac{\partial \chi^a(w)}{(z-w)}, & T_F(z) \chi^a(w) &\sim \frac{J^a}{(z-w)}. \end{aligned} \quad (2.11)$$

Finally there are two spin-field operators with conformal weight $\frac{3}{2}$:

$$C = Sp\{\chi^a\}_+ \quad \text{and} \quad S = Sp\{\chi^a\}_- \quad (2.12)$$

$$\begin{aligned} T_B(z) C(w) &\sim \frac{3C(w)}{2(z-w)^2} + \frac{\partial C(w)}{(z-w)}, & T_B(z) S(w) &\sim \frac{3S(w)}{2(z-w)^2} + \frac{\partial S(w)}{(z-w)} \\ J^a(z) C(w)_\alpha &\sim f_{bc}^a \sigma_{\alpha\beta}^{bc} \frac{C(w)_\beta}{(z-w)}, & J^a(z) S(w)_\alpha &\sim f_{bc}^a \sigma_{\alpha\beta}^{bc} \frac{S(w)_\beta}{(z-w)} \end{aligned} \quad (2.13)$$

where $\sigma_{\alpha\beta}^{ab}$ are the σ -matrices of $SO(24)$:

$$\sigma_{\alpha\beta}^{ab} = \frac{1}{2} [\gamma^a, \gamma^b]_{\alpha\beta}$$

The indices a, b are vector representation indices under $SO(24)$ and adjoint representation indices under H .

$$\begin{aligned} C(z)_\alpha C(w)_\beta &\sim \frac{\delta_{\alpha\beta}}{(z-w)^3} + \frac{\sigma_{\alpha\beta}^{ab} \chi_a \chi_b}{(z-w)^2} + \frac{\delta_{\alpha\beta} 2T_B + (\gamma^a \gamma^b \gamma^c \gamma^d)_{\alpha\beta} (\chi_a \chi_b \chi_c \chi_d)}{(z-w)}, \\ S(z)_\alpha S(w)_\beta &\sim \frac{\delta_{\alpha\beta}}{(z-w)^3} + \frac{\sigma_{\alpha\beta}^{ab} \chi_a \chi_b}{(z-w)^2} + \frac{\delta_{\alpha\beta} 2T_B + (\gamma^a \gamma^b \gamma^c \gamma^d)_{\alpha\beta} (\chi_a \chi_b \chi_c \chi_d)}{(z-w)}. \end{aligned} \quad (2.14)$$

$$C(z)_\alpha S(w)_\beta \sim \frac{1}{(z-w)^{\frac{1}{2}}} \left[\frac{(\gamma^a)_{\alpha\beta} \chi_a}{(z-w)^2} + \frac{(\gamma^a)_{\alpha\beta} \partial \chi_a + (\gamma^a \gamma^b \gamma^c)_{\alpha\beta} (\chi_a \chi_b \chi_c)}{(z-w)} \right] \quad (2.15)$$

The above OPE relations between $T_B, T_F, C_\alpha, J^a, \chi_a$ define a *new chiral superconformal algebra*. As we will see in the next section, the $C(z)S(w)$ OPE in Eq. (2.15) implies a boson-fermion Spectral Flows which guaranties the massive boson-fermion degeneracy of the Vacuum.

2.3 Spectral flows and the $MSDS$ operator-relations

The vertex operators are dressed by the super-reparametrization ghosts [21] [1]: the space-time boson vertices are expressed either in the zero or -1 ghost picture. The space-time fermions are in the $\pm\frac{1}{2}$ picture.

$$\mathbf{V}_b = e^{-\Phi} \chi_a, \quad \mathbf{S}_f = e^{-\frac{1}{2}(\Phi+iH_0)} S_\alpha \quad (2.16)$$

where the H_0 is the usual helicity field defined via bosonization of Ψ_0 and Ψ_L : $i\partial H_0 = \Psi_0\Psi_L$. The conformal weight h_q of the operator [21] [1]

$$e^{q\Phi} \longrightarrow h_q = -\frac{1}{2}q(q+2) \quad (2.17)$$

is such that \mathbf{V}_b has conformal weight $h_b = 1$ while \mathbf{S}_f has a weight $h_f = 2$. Thus, the string spectrum of bosons starts from a massless sector. Contrary, all space-time fermions are massive starting from the mass level 1. At the massless level, only bosons are present: namely those which are created from the left- and right-moving vacuum acting with the oscillators $\chi_{-\frac{1}{2}}^a$ and $\bar{\chi}_{-\frac{1}{2}}^a$:

$$\chi_{-\frac{1}{2}}^a |L> \otimes \bar{\chi}_{-\frac{1}{2}}^a |R> \quad (2.18)$$

At the massive level both bosons and fermions are present. At the first level we have the following states:

$$\left[\{\chi_{-\frac{3}{2}}^a \oplus \chi_{-\frac{1}{2}}^a \chi_{-\frac{1}{2}}^b \chi_{-\frac{1}{2}}^c\} \oplus Sp\{\chi^a\}_- \right] |L> \otimes \left[\{\bar{\chi}_{-\frac{3}{2}}^a \oplus \bar{\chi}_{-\frac{1}{2}}^a \bar{\chi}_{-\frac{1}{2}}^b \bar{\chi}_{-\frac{1}{2}}^c\} \oplus Sp\{\bar{\chi}^a\}_- \right] |R> \quad (2.19)$$

where $Sp\{\chi^a\}_-$ is the spin field of $SO(n)$, $n = 24$ with chirality “-” and conformal weight, $h = n/16 = 3/2$.

Thanks to the identity [20],

$$n + \frac{1}{6}n(n-1)(n-2) = \frac{1}{2}2^{\frac{n}{2}} = 2048 \quad \text{valid for } n = 24 \quad (2.20)$$

the degeneracy of *space-time bosons*:

$$\begin{aligned} & \{\chi_{-\frac{3}{2}}^a \oplus \chi_{-\frac{1}{2}}^a \chi_{-\frac{1}{2}}^b \chi_{-\frac{1}{2}}^c\} |L> \otimes \{\bar{\chi}_{-\frac{3}{2}}^a \oplus \bar{\chi}_{-\frac{1}{2}}^a \bar{\chi}_{-\frac{1}{2}}^b \bar{\chi}_{-\frac{1}{2}}^c\} |R> \\ & \oplus Sp\{\chi^a\}_- |L> \otimes Sp\{\bar{\chi}^a\}_- |R> \end{aligned} \quad (2.21)$$

and the degeneracy of *space-time fermions*:

$$\begin{aligned} & Sp\{\chi^a\}_- |L> \otimes \{\bar{\chi}_{-\frac{3}{2}}^a \oplus \bar{\chi}_{-\frac{1}{2}}^a \bar{\chi}_{-\frac{1}{2}}^b \bar{\chi}_{-\frac{1}{2}}^c\} |R> \\ & \oplus \{\chi_{-\frac{3}{2}}^a \oplus \chi_{-\frac{1}{2}}^a \chi_{-\frac{1}{2}}^b \chi_{-\frac{1}{2}}^c\} |L> \otimes Sp\{\bar{\chi}^a\}_- |R> \end{aligned} \quad (2.22)$$

are equal at least at the first massive level. This spectacular result is not a numerical accident of the first massive level but a deeper fundamental property of the affine $SO(24)$ algebra

which leads to the θ^{12} -identity in Eq. (2.9). The relation at the first massive level in Eq. (2.20) together with the OPE's of the previous section indicate the important role of the operator $O_{3/2}$, ($h_O = 3/2$):

$$O_{3/2} \equiv (\gamma^a)_{\alpha\beta} \partial\chi_a + (\gamma^a\gamma^b\gamma^c)_{\alpha\beta}\chi_a\chi_b\chi_c \equiv \partial\hat{\chi} + \hat{\chi}\hat{\chi}\hat{\chi}$$

($\hat{\chi}$ is a short hand notation for $\gamma^a\chi_a$ where γ^a are the γ -matrices of $SO(24)$). $O_{3/2}$ appears in the rhs of Eq. (2.15) and is used to define a massive bosonic vertex operator in (-1) ghost picture:

$$\mathbf{B}_b = e^{-\Phi} (\partial\hat{\chi} + \hat{\chi}\hat{\chi}\hat{\chi}). \quad (2.23)$$

\mathbf{B}_b has conformal dimension $h_B = 2$ and describes *massive bosonic states* starting at the first string level in a very similar way that \mathbf{S}_f describes *massive fermionic states* starting equally at the first string level. The spectral flow of \mathbf{B} -states to \mathbf{S} -states is express by the action of a ‘‘Spectral Flow Operator’’ \mathbf{C}_{sf} :

$$\mathbf{C}_{sf} = e^{\frac{1}{2}(\Phi+iH_0)} C$$

\mathbf{C}_{sf} is written in the (+1/2) ghost picture. It has conformal dimension $h_C = 1$ and +1/2 helicity charge. Thus, generically, \mathbf{C}_{sf} acting on ‘‘physical’’ bosonic states produces fermionic states (and vis versa). Although the \mathbf{C}_{sf} action looks like a space-time supersymmetry transformation, the actual situation turns out to be drastically different from that of supersymmetry. Indeed, the \mathbf{C}_{sf} action leaves invariant the massless bosonic states of the theory, therefore the boson-to-fermion mapping does not exist for the massless states. This statement is visualized in the OPE:

$$\mathbf{C}_{sf}(z) \mathbf{V}_b(w) \sim \mathbf{S}_f(w), \text{ finite for } z \rightarrow w \quad (2.24)$$

The absence of singular terms in $(z-w)$ shows clearly that the massless states are invariant under \mathbf{C} -transformation. On the other hand the \mathbf{C} -action on the massive states is not-trivial:

$$\mathbf{C}(z) \mathbf{B}(w) \sim \frac{\mathbf{S}(w)}{(z-w)} + \text{finite terms} \quad (2.25)$$

The above equation shows that the *massive bosonic states* are mapped to the fermionic ones. To prove the reverse action $\mathbf{C}(z) : \mathbf{S}(w) \rightarrow \mathbf{B}(w)$ it is not so direct since standard picture changing manipulation [21] are necessary to convert \mathbf{B} and \mathbf{S} to their conventional ghost-pictures. This conversion utilizes insertions of the ‘‘picture changing operator’’, of conformal

dimension zero, $e^\phi T_F$ [21] [2], in the OPE's. Modulo these manipulations, the reverse action follows mainly from the Eq. (2.15) and the picture changing equivalence of $J^a = f_{bc}^a \chi^b \chi^c$ in the (0) ghost picture with the $e^{-\phi} \chi^a$ in the (-1) ghost picture (see the OPE's in Eqs (2.11)).

Summarizing:

- $T_B, T_F, C_{3/2}$ and (J^a, χ^a) define via the OPE's a new super-conformal algebra.
- The closure of the algebra is guarantied when $c = 12$, so that $C_{3/2}$ is a chirality “+” spin-feld of $SO(24)$ with conformal weight $h_C = 3/2$.
- The realization of the algebra divides the “physical” states in two sectors :
 - i) Massless sector which is invariant under \mathbf{C} spectral flow transformations.
 - ii) Massive fermionic states \mathbf{S} with “-” chirality is in one to one correspondence with the massive bosononic states. $\mathbf{C} : \mathbf{S} \leftrightarrow \mathbf{B} \rightarrow \text{massive supersymmetry}$.

3 The heterotic $MSDS$ Vacua

In the right-moving sector of the heterotic string the $\mathcal{N} = 1$ supersymmetry is absent, while the left-moving degrees of freedom are as in the type II. The absence of the $2d$ conformal anomaly request 48 right-moving world-sheet fermions. Following the basic rules of the fermionic construction and respecting the left-moving gauge group H_L one can construct at least three distinguishable vacua according to the gauge group of the right-moving sector:

$$H_R = SO(48), \quad H_R = SO(32) \times E_8, \quad H_R = E_8^3 \quad (3.1)$$

In all three cases the heterotic partition function is tachyon free and factorizes in left- and right-holomorphic sectors:

$$Z_{Het}^{2d}(H_R) = \int_{\mathcal{F}} \frac{d^2\tau}{(\text{Im}\tau)^2} \left[\frac{(\theta_3^{12} - \theta_4^{12}) - (\theta_2^{12} - \theta_1^{12})}{2\eta^{12}} \right] \bar{\Gamma}[H_R] \quad (3.2)$$

where

$$\bar{\Gamma}[SO(48)] = \frac{1}{2} \left[\frac{(\theta_3^{24} + \theta_4^{24}) + (\theta_2^{24} + \theta_1^{24})}{\eta^{24}} \right] = 1128 + [j(\bar{\tau}) - 744]$$

$$\bar{\Gamma}[SO(32) \times E_8] = \frac{1}{2} \left[\frac{(\theta_3^{16} + \theta_4^{16}) + (\theta_2^{16} + \theta_1^{16})}{\eta^{16}} \right] \times \frac{1}{2} \left[\frac{(\theta_3^8 + \theta_4^8) + (\theta_2^8 + \theta_1^8)}{\eta^8} \right] = 744 + [j(\bar{\tau}) - 744]$$

$$\bar{\Gamma}[E_8^3] = \left\{ \frac{1}{2} \left[\frac{(\theta_3^8 + \theta_4^8) + (\theta_2^8 + \theta_1^8)}{\eta^8} \right] \right\}^3 = 744 + [j(\bar{\tau}) - 744] \quad (3.3)$$

The three constructions give different number of massless states coming from the right-moving sector. The right-moving massive states are expressed in terms of the unique holomorphic modular invariant function $j(\tau)$ of weight 24. The left-moving sector gives rise to *MSDS* symmetry which implies a constant contribution $C_L = 24$. Finally, the integrated partition function becomes:

$$Z_{Het}^{2d}(G_R) = \frac{\pi^2}{3} C_L \times C_R \quad (3.4)$$

with

$$C_L = 24, \quad C_R [SO(32) \times E_8] = C_R [E_8^3] = 744, \quad C_R [SO(48)] = 1128.$$

3.1 The type II Orientifold *MSDS* Construction

Another interesting example with *MSDS* structure is the based to type II orientifolds. When $H_L = H_R$ the type II partition function becomes a perfect square. This remark indicates how to define the orientifold projection $[8] \mathbb{Z}_2$:

$$\mathbb{Z}_2 \equiv \Omega. \quad (3.5)$$

As usual Ω interchanges the left- and right- movers. Following the rules of orientifold construction, the torus partition function of the close string sector, $\mathcal{T} = Z_H^{2d}$ counts with the invariant linear sum of a state under \mathbb{Z}_2 and as long as they are distinct. To complete the closed string states counting, one introduces the Klein bottle \mathcal{K} in which only \mathbb{Z}_2 invariant states appear. Each product of complex conjugate characters in \mathcal{T} descend to a character in \mathcal{K} , with argument $2i\tau_2$. At the end the Klein bottle amplitude has a very simple form thanks to the θ^{12} -identity:

$$2\mathcal{K} = \int_0^\infty \frac{d\tau_2}{\tau_2^2} [V^{(24)} - S^{(24)}] = \int_0^\infty \frac{d\tau_2}{\tau_2^2} C, \quad \text{with } C = 24 \quad (3.6)$$

It turns out that the annulus amplitude \mathcal{A} has the same form modulo the Chan-Paton degeneracies:

$$2\mathcal{A} = n^2 \int_0^\infty \frac{d\tau_2}{\tau_2^2} [V^{(24)} - S^{(24)}] = n^2 \int_0^\infty \frac{d\tau_2}{\tau_2^2} C, \quad \text{with } C = 24 \quad (3.7)$$

The factor n^2 is introduced to account for the Chan-Paton degeneracies. Finally the Mobius amplitude becomes:

$$2\mathcal{M} = -n \int_0^\infty \frac{d\tau_2}{\tau_2^2} [V^{(24)} - S^{(24)}] = -n \int_0^\infty \frac{d\tau_2}{\tau_2^2} C, \quad \text{with } C = 24 \quad (3.8)$$

As usual, the rank of the gauge group is determined by tadpole conditions for massless states, extracted from the transverse closed-string channel [8]. Clearly, this model is free of R-R tadpoles since the R-R sectors are massive, while the dilaton tadpole reads [8],

$$2\tilde{K} + 2\tilde{A} + 2\tilde{M} = \int_0^\infty dl (2 + \frac{1}{2}n^2 - 2n) C = 0, \quad \text{for } n = 2. \quad (3.9)$$

The above expressions shows clearly that the absence of any tadpoles from the open sector requires the choice of $n = 2$ and so, the gauge group in the open sector is fixed to be $SO(2)$. This situation is indeed extremely interesting since it implies that the total contribution of the open sector and not only the tadpoles vanishes identically! This remarkable fact is again a consequence of the θ^{12} -identity which implies that the integrant C is constant. Thanks to this fact, only the close sector gives non-trivial result in the partition function:

$$Z_{Orient}^{2d} = \frac{1}{2} Z_{II}^{2d} \quad (3.10)$$

4 Marginal deformation of the $MSDS$ vacua

In type II $MSDS$ vacua the massless bosons are $C_L \times C_R$ scalars. C_L and C_R are equal to the dimension of the adjoint representation of gauge group H_L and H_R respectively, ($C_L = C_R = 24$). These scalars parametrize the manifold in a similar way like the gauged supergravities:

$$\mathcal{K} = \frac{SO(C_L, C_R)}{SO(C_L) \times SO(C_R)}. \quad (4.1)$$

Because of the non-abelian structure of $H_L \times H_R$ the only marginal deformations are those that correspond to the Cartan sub-algebra $U(1)^{r_L} \times U(1)^{r_R}$, with r_L and r_R is the rank of H_L and H_R respectively. The moduli space of these deformations, $(M_{IJ} J_L^I \times J_R^J)$, is reduced to:

$$\mathcal{M} = \frac{SO(r_L, r_R)}{SO(r_L) \times SO(r_R)}. \quad (4.2)$$

The maximal number of the moduli M_{IJ} is when:

$$H_L = H_R = SO(4)_{k=1}^4 = SU(2)_{k=2}^8, \quad \text{with } r_L = r_R = 8. \quad (4.3)$$

In that precise case, the integrand of the deformed partition function factor out *a shifted lattice* $\Gamma_{8,8}(M) \left[\begin{smallmatrix} h_i \\ g_i \end{smallmatrix} \right]$; The latter couples non-trivially to the “parafermion numbers” [22] [3] defined by the gauge WZW-cosets [3]:

$$\prod_{I_L=1,\dots,8} \left(\frac{SU(2)_{k=2}}{U(1)} \right)_{I_L} \times \prod_{I_R=1,\dots,8} \left(\frac{SU(2)_{k=2}}{U(1)} \right)_{I_R}$$

The fact that the level k is equal to $k = 2$, makes the above coset structure to be equivalent to eight left-moving world sheet fermions, Ψ_{I_L} and eight right-moving ones, Ψ_{I_R} . At the end, the shifted $\Gamma_{8,8}$ lattice [23] couples non-trivially to the left- $\{\Psi_{I_L}\}$ and right- $\{\Psi_{I_R}\}$ R -symmetry charges [24] of the conventional type II superstrings ! Indeed, in the large moduli limit (modulo S, T, U -dualities), the $\Gamma_{8,8}$ lattices decompactifies and the correlations with the R -symmetry charges become irrelevant [24]. At this limit one recovers the conventional ten dimensional type II supersymmetric vacua ! For large but not infinitely large deformations, the obtained vacua are those of “spontaneously broken supersymmetric vacua in the presence of geometrical fluxes” [10] studied in details refs [24] [25]. Furthermore, some of the Euclidian version of the models, correspond to “thermal stringy vacua” in the presence of non-trivial left-right asymmetric “gravito-magnetic fluxes” studied recently in refs [26] [25]. The would be “initial” classical singularity of general relativity as well as the stringy Hagedorn-like singularities are both resolved by these fluxes ! [26]

The above generic properties of the deformed $MSDS$ vacua, strongly suggest the following **Cosmological Conjecture**:

- The $MSDS$ vacua, (or even less symmetric orbifold versions of those), are potential candidates able to describe *the early non-singular phase of a stringy cosmological universe*.
- During the cosmological evolution the deformation moduli $M_{IJ} \rightarrow M_{IJ}(t)$ evolves with the time. Once $M_{IJ}(t)$ are sufficiently large (modulo S, T, U -dualities), an effective field theory description emerges with an induced “space-time geometry” of an *effective higher dimensional space-time*. Also, the relevant degrees of freedom and interactions are well described by some “no-scale” effective supergravity theories [27] of the conventional superstrings [28].

- The effects of the initial *MSDS* structure induces at the large moduli limit non-trivial “geometrical” fluxes [10], [24] [26] which in the language of the effective supergravity give rise to a spontaneous breaking of supersymmetry [10] [16] and to a finite temperature description of the effective theory [16] [25] [26]

The above discussion still valid for the other choices of H_L, H_R . The rank however is reduces:

- (i) $r = 4$ for $H = SU(5)$, $H = SO(7) \times SU(2)$ or $H = G_2 \times Sp(4)$,
- (ii) $r = 6$ for $H = SU(4) \times SU(2)^3$ or $H = SU(3)^3$.

In the large M_{IJ} -moduli limit of the deformed $H = SU(2)^8$ vacua, the effective geometric description would be up to 10-dimensional space-time. In the lower rank cases however, the effective space-time cannot be higher than 6-dimensional in the case (i) and not higher than 8-dimensional in the case (ii).

The Heterotic *MSDS* vacua contain much more massless states. Here $C_R = 744$ when $H_R = SO(32) \times E_8$ or $H = E_8^3$ and $C_R = 1128$ when $H_R = SO(48)$. In all cases however the dimension of the right-moving gauge group is $C_R = 24$. The previous properties of the deformed *MSDS* type II vacua, still valid in the heterotic and in the orientifold deformed *MSDS* vacua.

5 Further perspectives and conclusions

The existence of a new of massive boson-fermion degeneracy symmetry, is shown by explicit construction in the framework of Heterotic, Type II and Type II orientifold superstring theories. In all constructions the target space-time is 2-dimensional and their spectrum consist of massless bosonic degrees of freedom while all massive boson and fermion degrees of freedom show a new Massive Spectrum Degeneracy Symmetry *MSDS*. This remarkable property follows from the modular properties between the Vector (V), Spinor (S) and Anti-Spinor (C) characters of the affine $SO(24)$ algebra that are formulated algebraically in terms of θ^{12} -identity, Eq. (2.9).

A new chiral $N = 1$ superconformal algebra is proposed based on the usual $N = 1$ super-

Virassoro operators, T_B , ($h_B = 2$) and T_F , ($h_F = 3/2$) together with C , ($h_C = 3/2$) and J^a , ($h_J = 1$), with J^a are the currents of a semi-simple gauge group H , of dimension $d_H = 24$. A simple realization of this new algebra is given in terms of world-sheet fermions. The massive boson-fermion degeneracy follows from a “spectral flow” relations induced by the algebra $\{T_B, T_F, C, J^a\}$.

In all *MSDS*-vacua, the non-abelian gauge group H is unbroken. Marginal deformations of current-current type, $M_{IJ} J_L^I \times J_R^J$ breaks *spontaneously* H_L and H_R to abelian sub-groups $U(1)^{r_L} \times U(1)^{r_R}$. What is extremely interesting is the fact that in the *large M_{IJ} deformation limit* the strongly-deformed *MSDS*-vacua, are well described in terms of an effective “higher dimensional” conventional superstring theory in which the space-time supersymmetry is *spontaneously broken* by “geometrical” and “thermal” fluxes. This fundamental generic properties of the deformed *MSDS*-vacua, strongly suggest to consider them as the most (semi-) realistic candidates able to describe the “early non-singular phase of our Universe”, free of initial “general relativity singularity” and free of any “Hagedorn-like” stringy singularities. Further investigations in this direction are necessary, and will be exhaustively studied by the author and collaborators in future publications.

Another noticeable property of the 2-d *MSDS*-vacua is the *holomorphic factorization* property of their partition function. Although these theories have non trivial massive spectrum, thanks to *MSDS*, all non-topological degrees of freedom are effectively washed out from the partition function ! In that respect, *MSDS*-vacua realize 2-d target-space conformal field theories with the holomorphic factorization properties similar to those initially proposed by Witten [29] in the context of BTZ black holes [30]. In this context 2-d *MSDS*-vacua (mainly the heterotic ones), are identified with the boundary 2-d conformal field theory of *AdS*₃ [30]. Following Witten’s conjecture, the massive bosonic spectrum is identified with the mass spectrum of BTZ-black-holes [29]. The *MSDS* theories however, suggest more, mainly, the existence of a “massive supersymmetry” fermionic partner having the same mass spectrum as the bosonic one !

The other interesting structure of *MSDS*-vacua, is their connection to the “gauge supergravity theories”. Although this connection is **not yet** well transparent in the undeformed

$MSDS$ -vacua, (where the description is non-geometrical), it is well established however in the “strongly deformed phase” via the induced geometrical fluxes of the effective higher dimensional theories.

Finally, an interesting question is whether it is possible to construct in a higher than two dimensions a field theory with an unbroken $MSDS$, and in particular in $d = 4$. A progress in that direction may give an alternative to the conventional supersymmetry approach concerning the well known mathematical inconsistencies related to the hierarchy and to the cosmological constant problem.

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